Nonlinear Finite Element Analysis of a Laminated Cylindrical Shell with Transverse Matrix Cracks

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A clamped laminated cylindrical shell is presented to investigate nonlinear structural behavior involving geometrically nonlinear deformation. In the investigation, transverse matrix cracks are considered in the stiffness of the laminated cylindrical shell. Stiffness degradation is examined for several laminated angles and transverse crack density. Micro-mechanics theory on the composite material was used to derive the degraded stiffness of the laminated cylindrical shell due to the crack density. Iterative numerical scheme was developed to calculate the degraded composite stiffness which is a complicated relation with the crack density. A nonlinear finite element program was developed using 3-D degenerated shell element and the first order shear deformation theory to consider the large deformation of the clamped laminated cylindrical shell. The updated Lagrangian method is used for nonlinear finite element analysis. Nonlinear structural responses of the laminated cylindrical shell were examined for various stacking sequences and crack density under transversely loaded pressure. Also, the effect of crack opening/closed was considered in the examination. Through this study, it is realized that the transverse matrix crack causes moderate stiffness reduction and affects the responses of the composite shell.

Key Words : Laminated Cylindrical Shell, Nonlinear Finite Element Analysis, Transverse Matrix Crack, Stiffness Degradation

1. Introduction

Laminated composite panels, applied by bending and twisting moments, have been investigated for studying the effect of transverse matrix cracks. Previous works have been studied for the crack itself, which may affect the behaviour of laminated composite structures.

The matrix crack cause stiffness degradation in the composite structures. Therefore, the stiffness degradation due to the crack is main cause of the collapse in laminated structures.

Highsmith et al. (1982) predicted the stiffness degradation of a cross-ply laminate including the matrix cracks using shear lag analysis method. Flaggs (1985) extended the method for the two dimensional shear lag analysis. Lim, et al. (1989) predicted the onset of the transverse cracks considering interlaminar shear layer. Lee, et al. (1990) proposed a simplified shear lag analysis using a progressive damage scheme for the crossply composite laminate under uniaxial tensile loading. The shear lag analysis and probabilistic analysis have been taken advantage of by several researchers (Wang et al., 1984; Laws and Dvorak, 1988; Fukunaga et al., 1984).

Nuismer and Tan (1988) proposed the analysis of progressive matrix cracking in the composite laminate, based on the two dimensional elasticity theory. Talreja (1965) predicted stiffness reduction due to the transverse cracking during crack growth using predeveloped stiffness damage relationship. Laws and Dvorak (1983; 1985) inves-

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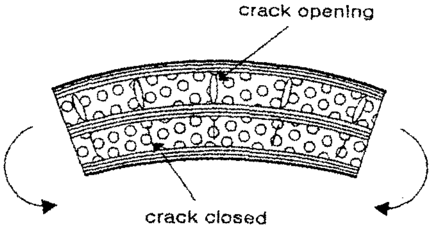


Fig. 1 Crack opening and closing under transverse load.

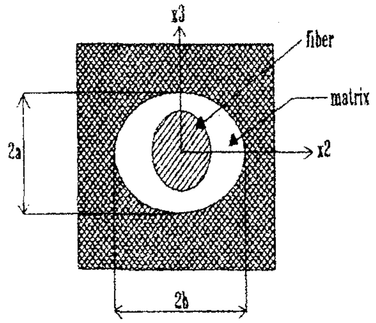


Fig. 2 Doubly embedded fiber and matrix.

tigated stiffness reduction using self consistent method and deriving stress-strain relation in the presence of matrix cracks.

Most of the previous works have been investigating stiffness reduction due to the transverse matrix crack in a curved composite laminate under two dimensional or plane loading. In recent years, stiffness reduction of a curved composite laminate panel has been examined under bending or twisting moments (Park, Kang, Chung, and Lee, 1997; Park and Lee, 1998).

In present study, nonlinear structural behavior of laminated cylindrical shell will be examined under transversely loaded pressure by nonlinear finite element analysis. In the study, the effect of the crack density is included and crack opening/closing (Fig. 1) also considered. The numerical study is pursued for various laminate angles, stacking sequence and crack density.

2. Governing Equation for Composite Laminate in Two Phase System

The equations for a two-phase model can be derived using the theory by Hill (1965a; 1965b). The theory is based on the solution of an inclusion problem for an elliptic cylinder in an anisotropic elastic medium. The constitutive equations are

$$\sigma=C\varepsilon,\ \ \varepsilon=S\sigma,\ \ CS=SC=I \qquad (1)$$

$$Q=C-CPC \qquad (2)$$

where C and S are the fourth order stiffness and compliance tensors, respectively.

The geometry of the elliptic cylinder in an infinite homogeneous solid (Fig. 2) is

$$x_2^2/b^2+x_3^2/a^2=1 \ \ |x_1|<\infty \qquad (3)$$

For the inclusion, the properties of the surrounding medium are identically used. The solution of the problem need to determine the tensor P of which components can be referred in previous work (Kinoshita and Mura, 1971).

$$P_{ijkl}=\frac{ab}{8\pi}\int_0^{2\pi}\frac{(\omega_i f_{jk}\omega_l+\omega_j f_{ik}\omega_l+\omega_i f_{jl}\omega_k+\omega_j f_{il}\omega_k)}{(a^2\omega_1^2+b^2\omega_2^2)}d\psi \qquad (4)$$

where $\omega_1=\cos\psi$, $\omega_2=\sin\psi$ and f_{ik} are the inverse of $C_{ijkl}\ \omega_j\ \omega_l$. In this study, C and S are obtained from micromechanics relations, self-consistent method (Whitney and McCullough, 1990).

It is often convenient to work with the tensor Q which is defined by

$$Q=C-CPC \qquad (5)$$

Consider now an elliptic cylinder, with stiffness C_r and compliance S_r, which is embedded in an infinite matrix whose stiffness and compliance tensors are, respectively, C and S. The matrix is loaded by uniform stress, $\bar{\sigma}$, or subjected to uniform strain, $\bar{\varepsilon}$, at infinity. Let the stress and strain fields in the inclusion be σ_r and ε_r respectively, so that

$$\sigma_r=C_r\varepsilon_r,\ \ \varepsilon_r=S_r\sigma_r \qquad (6)$$

It is well known that the elastic field in the ellipsoidal inclusion is uniform and can be evaluated as

$$\begin{aligned}\varepsilon_r&=[I+P(C_r-C)]^{-1}\bar{\varepsilon}\\ \sigma_r&=[I+Q(S_r-S)]^{-1}\bar{\sigma}\end{aligned} \qquad (7)$$

Turning now to the basic equations for composites, we note that in order for the concept of overall moduli to be meaningful, it is essential to

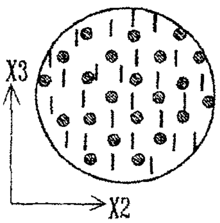


Fig. 3 Cross section of composite laminate in the two-phase.

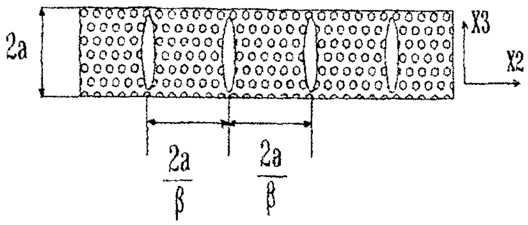


Fig. 4 A laminate with small diameter fibers and slit crack.

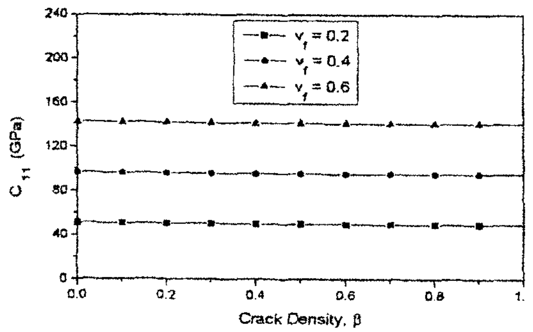


Fig. 5 Stiffness C_{11} vs crack density β.

consider macroscopically uniform loading. In such circumstances the applied stress is equal to the average stress, $\bar{\sigma}$, and the phase average stresses, $\bar{\sigma}_r$, and strains, $\bar{\varepsilon}_r$, are related to the overall averages through

$$\bar{\varepsilon}_r = A_r \bar{\varepsilon}, \quad \bar{\sigma}_r = B_r \bar{\sigma} \tag{8}$$

Suppose that parallel slit cracks are equally distributed in the composite and have the same direction with the fibers. In the phase-two medium, the fiber diameters are very small compared with the crack length or ply thickness 2 a. We choose x_1-axis to be normal to all crack planes, as shown in the Fig. 3. The cracks can be modelled by taking the aspect ratio in the elliptic cylinder to be zero. We will choose phase 2 (matrix) to contain voids and taking the limit $\delta \rightarrow 0$ of cracks. The overall stiffness and compliance tensors for the phase-two medium are obtained by aspect ratio $\delta = b/a$, the number of voids per unit area in $x_2 - x_3$ plane, and crack density $\beta = 4a^2\eta$.

$$C = C_u - \frac{1}{4}\pi\beta C_u \Lambda C \tag{9}$$

$$S = S_u + \frac{1}{4}\pi\beta\Lambda \tag{10}$$

In Eqs. (9)~(10), the subscript u denotes uncracked state. In two phase, parameter Λ is defined in Law and Dvorak's work (1988, 1983). Stresses are represented in vector form such as $\{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{13}\}^T$.

Nonzero parametric tensor Λ has only three components and α_1, α_2 are solution of quadratic Eq. (12).

$$\Lambda_{22} = \frac{C_{33}(\alpha_1^{1/2} + \alpha_2^{1/2})}{(C_{22}C_{33} - C_{23}^2)}$$

$$\Lambda_{44} = \frac{1}{4(C_{44}C_{66})^{1/2}} \tag{11}$$

$$\Lambda_{55} = \frac{(C_{22}C_{33})^{1/2}(\alpha_1^{1/2} + \alpha_2^{1/2})}{4(C_{22}C_{33} - C_{23}^2)}$$

$$C_{33}C_{55}\alpha^2 - (C_{22}C_{33} - C_{23}^2 - 2C_{23}C_{55})\alpha + C_{22}C_{55} = 0 \tag{12}$$

The stiffness considering the crack density parameter β for the cracked fibrous composite is obtained through iterative scheme. The Eq. (5) can be rewritten for numerical calculation in Eq. (13).

$$C = [I + \frac{1}{4}\pi\beta C_u \Lambda(C)]^{-1} C_u \tag{13}$$

The tensor Λ depends on the stiffness C. When we start iteration, the $\Lambda(C)$ is calculated by uncracked stiffness $C = C_{(u)}$. Iterations perform until convergence condition (14) is satisfied for the stiffness. The condition used herein is

$$(\|C^{(k+1)} - C^{(k)}\|)/\|C^{(k)}\| \leq \varepsilon \tag{14}$$

The error bound ε used here is 0.001. The stiffness C for the cracked fibrous composite has nine independent coefficients different from five constants for the uncracked composite.

3. Formulation for Nonlinear Finite Element Analysis

Equation for the (n+1)th equilibrium state is derived using the principle of virtual work (Bathe, 1982).

$$\int_{V^{n+1}} S_{ij}^{n+1}\delta\varepsilon_{ij}^{n+1}dV=\int_{V^{n+1}} f_i^{n+1}\delta u_i^{n+1}dV + \int_{S^{n+1}} T_i^{n+1}\delta u_i^{n+1}dS \qquad (15)$$

In Eq. (15), S_{ij}, ε_{ij}, f_i, T_i, and u_i denote the second Piola-Kirchhoff stress, Lagrangian strain, externally applied body force, externally applied surface traction, and displacements, respectively. V and S are volume and surface at the (n+1)th equilibrium state. Using the updated Lagrangian formulation (Bathe, 1982), the linearized equation of Eq. (15) for n-th equilibrium state is given in the Eq. (16).

$$\int_{V^n} C_{ijkl}^n\, e_{kl}\,\delta e_{ij}dV+\int_{V^n}\sigma_{ij}^n\delta\eta_{ij}dV=\delta W^{n+1} - \int_{V^n}\sigma_{ij}^n\delta e_{ij}dV \qquad (16)$$

In Eq. (16), C_{ijkl}, σ_{ij}, δW represent stiffness, Cauchy stress, and the external virtual work respectively. Incremental linear strain, e_{ij} and incremental nonlinear strain, η_{ij} are given in Eqs. (17)-(18).

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \qquad (17)$$

$$\eta_{ij}=\frac{1}{2}u_{k,i}u_{k,j} \qquad (18)$$

In Eqs. (17) and (18), incremental displacements u_i is defined as displacements at the n-th and (n+1)th equilibrium states in Eq. (19).

$$u_i=u_i^{n+1}-u_i^n \qquad (19)$$

Based on the theory of the first order shear deformation, the incremental displacement of the eight node three dimensional degenerated shell element (Panda and Natarajan, 1981; Chao and Reddy, 1984) is given by Eq. (20).

$$u_i=\sum_{k=1}^{8}N^k(\xi,\ \eta)\,\bar{u}_i^k + \frac{1}{2}\sum_{k=1}^{8}N^k(\xi,\ \eta)\,t^k\zeta V_i^k \qquad (20)$$

In Eq. (20), ξ, η, ζ are local coordinate of an element. $\bar{u}^k$, N^k, t^k, and V^k are incremental displacements, shape function, shell thickness and incremental normal vector in the k-th node, respectively. Using the Eqs. (17) ~ (20), the relations between strain and displacements are given by Eq. (21).

$$\mathbf{e}=\mathbf{B}_L\bar{\mathbf{u}},\quad \eta=\mathbf{B}_{NL}\bar{\mathbf{u}} \qquad (21)$$

In Eq. (21), $\mathbf{B}_L$ and $\mathbf{B}_{NL}$ are linear and nonlinear strain-displacement matrix, respectively. Using the modified Newton-Raphson iterative method (Cheney and Kincaid, 1985), Eq. (16) can be written by Eqs. (19) ~ (21) as follows.

$$(\mathbf{K}_L+\mathbf{K}_{NL})\Delta\bar{\mathbf{u}}^{(i)}=\mathbf{R}^{(i)}-\mathbf{F}^{(i-1)} \qquad (22)$$

where

$$\mathbf{K}_L=\int_{V^n}\mathbf{B}_L^T\mathbf{C}\ \mathbf{B}dV$$

$$\mathbf{K}_{NL}=\int_{V^n}\mathbf{B}_{NL}^T\sigma\mathbf{B}dV$$

$$\mathbf{F}=\int_{V^n}\mathbf{B}_L^T\hat{\sigma}dV$$

$$\mathbf{R}=\int_{V^n}\mathbf{N}^T\mathbf{f}dV+\int_{V^n}\mathbf{N}^T\mathbf{T}\ dS$$

are given. σ and $\hat{\sigma}$ are Cauchy stress matrix and Cauchy stress vector. $\mathbf{f}$ and $\mathbf{T}$ denotes body force and surface traction, respectively.

4. Calculation of Degraded Stiffness of a Laminated Shell

For the numerical study of composite laminated structures, elastic modulus and Poisson's ratio are calculated for the composite laminate including matrix cracks and stiffness matrix is obtained from the composite material properties. For the uncracked stiffness matrix is calculated in accordance with the crack density in matrix and fiber volume fraction, v_f. The cracked stiffness is obtained from the Eq. (13) to satisfy the convergence criteria (14).

In the finite element analysis, we have to decide whether the crack is open or closed. When the transverse stress σ_{22} at the crack is positive, the crack is considered as open; when the stress σ_{22} is negative, the crack is considered as closed. The crack opening/closed take effect on the constitutive relation. For crack opening, stiffness terms in the constitutive relation, corresponding to the transverse direction, are set to be zero. For crack closed, the stiffness is unchanged from the cracked stiffness, Eq. (13).

In practical analysis, the laminates containing matrix cracks are not able to resist tensions perpendicular to the fiber axis, but capable to do compressions. Therefore, the preliminary analysis, which calculates cracked stiffness and considers the crack opening/closed, will be proceeded again to judge whether the transverse stress in the laminate is positive or negative, until the state of transverse stresses becomes negative. After the preliminary analysis, nonlinear finite element analysis will start to iterate as explained in Eq. (22)

5. Numerical Example

5.1 Stiffness degradation of a composite material containing matrix crack

For several fiber volume fractions, (V_f=0.2, 0.4, 0.6), the degraded stiffnesses, C_{ij}, of a composite material are shown in Fig. 6~Fig. 13 for varying crack density, β. The stiffnesses are calculated using Eq. (13) until satisfying the convergence criteria (14). The material properties, used in the calculation, are listed in the Table 1. As shown in the Fig. 6~Fig. 13, stiffness in the fiber axis (C_{11}) remains almost constant. Other stiffnesses (C_{12}, C_{22}, C_{23}, C_{33}, C_{44}, C_{55}, C_{66}) decrease rapidly as the crack density increase. The other remaining stiffnesses show moderate change.

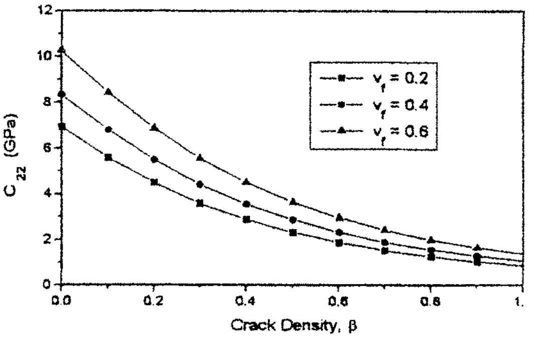


Fig. 6 Stiffness C_{22} vs crack density β.

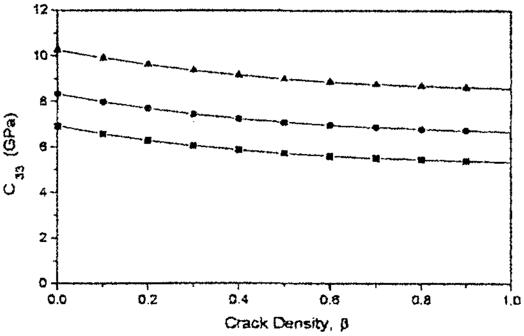


Fig. 7 Stiffness C_{33} vs crack density β.

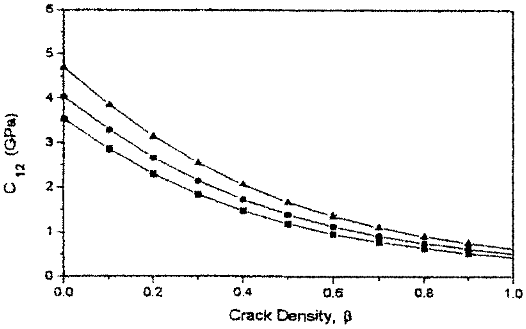


Fig. 8 Stiffness C_{12} vs crack density β.

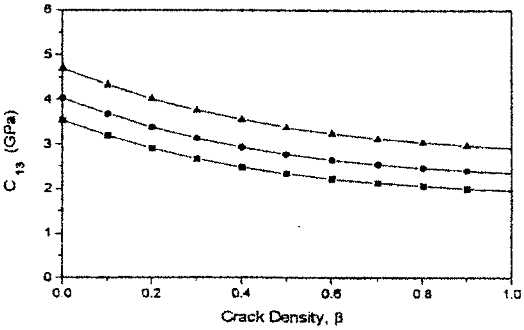


Fig. 9 Stiffness C_{13} vs crack density β.

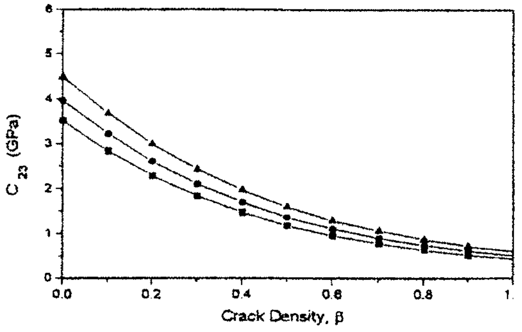


Fig. 10 Stiffness C_{23} vs crack density β.

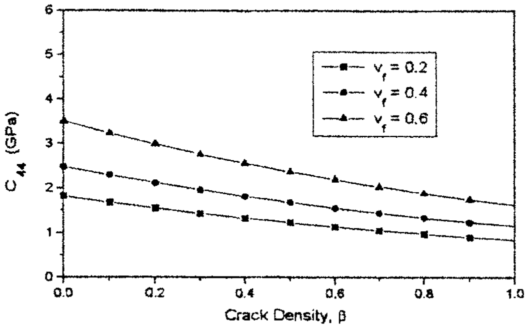


Fig. 11 Stiffness C_{44} vs crack density β.

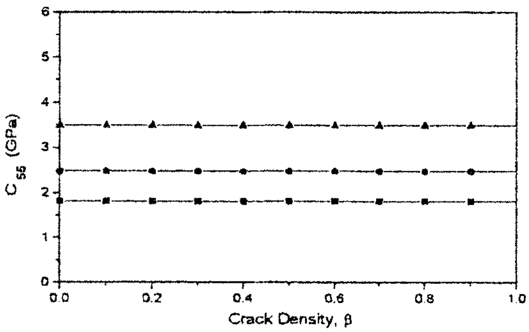


Fig. 12 Stiffness C_{55} vs crack density β.

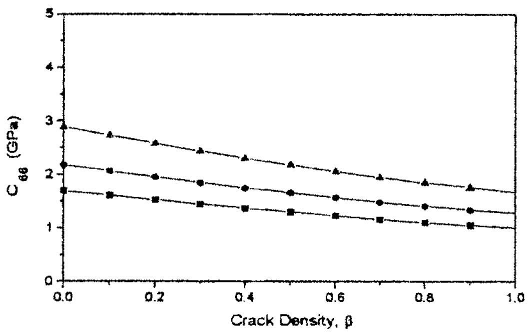


Fig. 13 Stiffness C_{66} vs crack density β.

Table 1 Material properties of T 300/976 Gr/Ep (GPa).

	E_{11}	E_{22}	G_{12}	G_{23}	ν_{12}
Fiber	230.0	16.0	9.0	6.1	0.3
Matrix	3.7	3.7	1.4	1.4	0.35

5.2 Verification of the nonlinear finite element program

In order to verify the nonlinear finite element analysis code used in this study, isotropic cylindrical shell is presented to compare to previous work which has been done by Crisfield (1981). The numerical model for the verification is shown in the Fig. 14. For finite element discretization, a 1/4 model of the cylindrical shell, which consists of 25 finite elements, is suggested taking advantage of the reflective symmetry. The present nonlinear finite element code is implemented using 8-node three dimensional degenerated shell element and the theory of the first order shear deformation. Reduced $2\times2\times2$ integration rule is adapted for numerical integration. The elastic modulus and Poisson's ratio are E=3,105 Mpa and ν=0.3. The radius of curvature and longitudinal length are R=B=2540 mm. The shell thickness is t=3.175 mm.

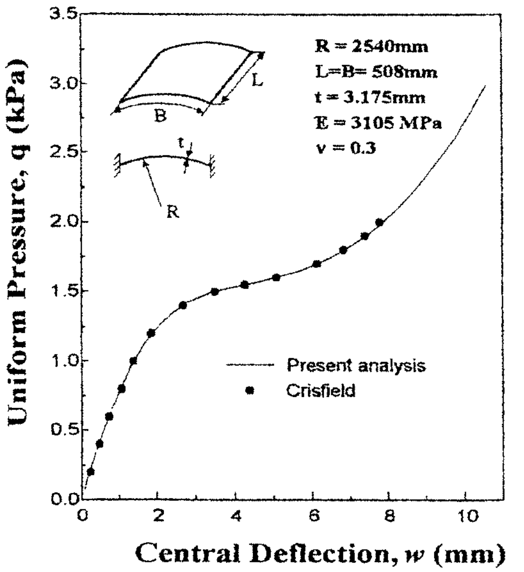
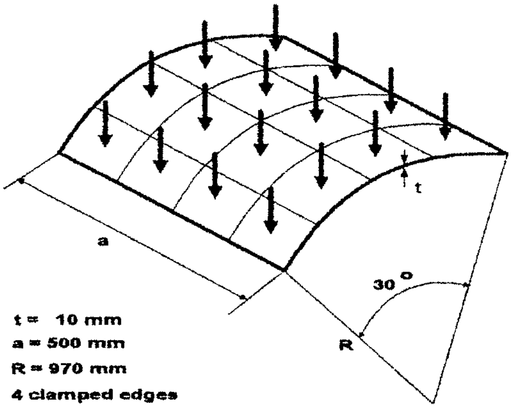


Fig. 14 Deflection at the center point of the cylindrical shell under transverse pressure.

In Fig. 14, present and Criesfield numerical results (1981) are plotted. Figure 14 shows the present nonlinear analysis code well matches to Criesfield work. Therefore, the present code, which is able to do nonlinear finite element analysis of laminated shell structures, is proved to handle isotropic cylindrical shell well. It is verified as usable to perform nonlinear finite analysis of a clamped cylindrically laminated shell.



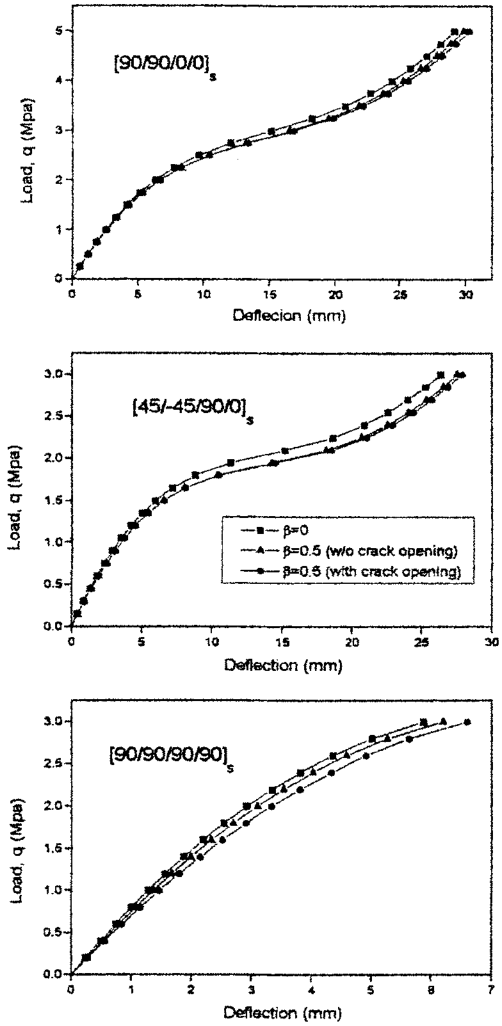


Fig. 15 Distributed load vs. deflection at the center of the laminated shell.

5.3 A cylindrically laminated shell clamped at 4 edges

The cylindrically laminated shell is clamped at 4 edges and applied by transversely loaded uniform pressure. In the finite element model, 16 elements are used. The deflections are examined at the center of the cylindrical shell.

The deflections are varied for several laminated sequences ($[90/90/0/0]_s$, $[45/-45/90/0]_s$, and $[90/90/90/90]_s$) and crack densities ($\beta=0$; $\beta=0.5$ with/without crack opening/closed) as shown in the Fig. 15. Fig. 16 shows deflections by changing the crack density from 0 to 1 without considering crack opening/closed effect. The analysis provides the fact that the deflections considering large deformation in the nonlinear structural analysis vary moderately by increasing crack density. Fig. 17 is the plot for deflections varying the crack density for three laminated sequences ($[45/-45/90/0]_s$, $[90/90/0/0]_s$, and $[0/0/0/0]_s$).

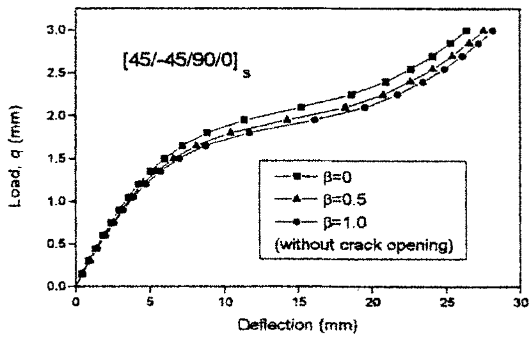


Fig. 16 Distributed load vs. deflection varying crack density.

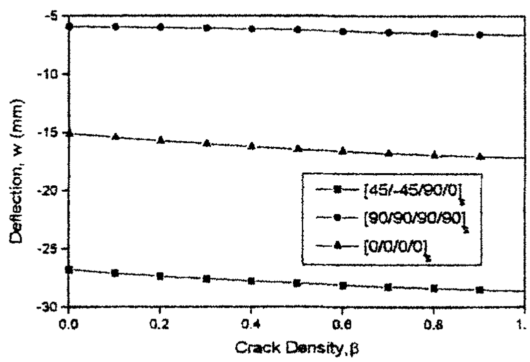


Fig. 17 Deflection vs crack density β.

6. Conclusion

In this study, considering geometrically nonlinear deformation, the stiffness degradation of laminated cylindrical shell was investigated in the variation of crack density and fiber volume fraction. After the crack opening/closed has been decided by checking the transverse stresses in the cylindrical shell, nonlinear finite element analysis has been done. The stiffness degradation of a composite material was investigated. As a numerical example, a laminated cylindrical shell clamped at 4 edges was applied by transverse uniform pressure.

From the numerical results, it was shown that the consideration of crack opening/closed affects on the deflections of the laminated cylindrical shell containing matrix cracks, resulting from the stiffness reduction of composite laminate. The crack opening/closed should be included for the study of composite structures including nonlinear deformation.

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